Name: _____

Calculus I

Professor Piotr Hajłasz

First Exam

October 3, 2014, 11:00-11:50am.

Problem	Possible points	Score
1	20	
2	30	
3	20	
4	10	
5	20	
Total	100	

Exercise 1. (20p) Find the following limits

(a)

$$\lim_{x\to 5} \frac{(x-5)\sin\left(\frac{\pi x}{20}\right)}{\sqrt{x+4}-3}\,.$$

(b)

$$\lim_{x\to\infty} \frac{\sqrt{2x^4+1}+\sin(2x)}{x^2+25x+2014}.$$

Exercise 2. (30p)

(a) For what value of a is the function continuous

$$f(x) = \begin{cases} \frac{6x^2}{\sin(x^2)} & \text{if } x > 0, \\ (a^2+1)\sin x + 3a & \text{if } x \leq 0. \end{cases}$$

(b) Use the definition to find the derivative of $f(x) = \sqrt{3x + 5}$.

(c) Find vertical and horizontal asymptotes of $f(x) = \dfrac{(x-2)\sin(x^2+1)}{x^3-5x^2+6x}$.

Exercise 3.(20p) Find the derivative of
(a)

$$\cos\left(\sqrt{x+\sin\left(\frac{1}{\sqrt{x+1}}\right)}\right).$$

(b)

$$\frac{\cos^2 x - x}{2 + \tan x}\,.$$

Exercise 4.(10p) Find dy/dx if $y^2 = x^2 + \sin(xy)$.

Exercise 5. (20p) A girl files a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?